RESPONSE SURFACE DESIGNS FOR CONDUCT OF AGRICULTURAL EXPERIMENTATION

By

B.S. Gill[1] and M.N. Das[2]

(Received in February, 1973; Accepted in July, 1973)

Introduction

Box and Hunter (1957) introduced a class of factorial designs which are known as Rotatable designs. These designs have since been applied extensively in the fields of industry and chemical engineering. There has not been much application of these designs for agricultural field experimentation.

The main difficulty in the way of application of rotatable design for agricultural field experimentation is that almost no work is available in literature where such designs are presented broken into blocks of equal size. Though Box and Hunter have given some second order designs with blocking, the block sizes in all these designs are unequal. In experiments with field plots as the experimental units intra-block error depends on the block size. Das and Gill (1973), therefore, give a procedure of forming blocks of rotatable designs of equal size and at the same time not very large.

In this paper several designs with 3 and 4 factors either in three or five levels have been constructed. The designs have been obtained first in terms of coded doses with suitably chosen origin and scale. These doses are required for fitting the response surface. For the actual layout it is desirable to present the block contents in terms of the actual doses to be used. Each design has thus been presented in terms of both these dose symbols. These designs ase given in the appendix.

2. Requirements of Second Order Rotatable Designs

A set of N combinations of K variates x_1, x_2, x_3..............x_k with suitable origin and scale, each representing the levels of a factor

1. Senior Statistician (SC), Ministry of Agriculture, New Delhi.

2. Director, Institute of Agricultural Research Statistics, New Delhi.

to be called hereafter, N points of a design or simply design points and 'y' denoting the responses obtained from different combinations of the x-variates, will from a Rotatable design of order d if the surface

$$Y=\beta_0+\Sigma\beta_i x_i+\Sigma\Sigma\beta_{ij}x_i x_j+\Sigma\Sigma\Sigma\beta_{ijk}x_i x_j x_k+\ldots\ldots\ldots\ldots\text{up power } d$$

can be so fittted with the data collected from these N points that the variance of the response at any point, say, $(x_{10}, x_{20}, x_{30}, \ldots\ldots x_{ko})$ estimated through the response surface is a function of the distance of the point form the origin. A rotatable design will satisfy the following condition :

(A) :

$$\Sigma x_i=0$$
$$\Sigma x_i x_j=0$$
$$\Sigma x_i x_j^2=0,\ \Sigma x_i^3=0,\ \Sigma x_i x_j x_k=0$$
$$\Sigma x_i x_j x_k x_l=0,\ \Sigma x_i x_j^3=0,\ \Sigma x_i x_j x_k^2=0$$

for $i\neq j\neq k\neq l.$

(B) :

$$\Sigma x_i^2=\text{constant}=N\lambda_2$$
$$\Sigma x_i^4=\text{constant}=3N\lambda_4$$
$$\Sigma x_i^2 x_j^2=\text{constant}$$

(C) :

$$\Sigma x_i^4=3\ \Sigma_i^2 x_j^2$$

(D) :

$$\frac{\lambda 4}{\lambda_2^2}>\frac{k}{k+2}$$

Each of the summation in the above relations is over the design points. For example, Σx_i denotes the summation of all the value of the i-th variate, x_i in the N points.

If necessary without loss of generality we can take λ_2 to be unity for providing standard scales to the variates. If the design points are to be distributed into several blocks, the following further conditions should also be satisfied for estimating the constants in the response surface free from block effects.

(*i*) $\Sigma x_i=0$ constant within each block.

(*ii*) $\Sigma x_i x_j=0$ within each block.

and (*iii*) $\dfrac{\Sigma_m x_i^2}{\Sigma_1 x_i^2}=\dfrac{n_m}{n_1}$ for each pair of blocks.

Here Σ_m denotes summation over the points in the m-th block, Σ_1 having similar meaning; n_m and n_1 denote the number of points in the m-th and i-th blocks respectively.

3. Relationship between Coded and Actual Doses

The relationship between the actual doses and the coded doses is described below. When there are three levels of a factor they are always equispaced in this deviation and would be coded as $-a$, 0, a and the corresponding level of actual doses would be 0, $\frac{M}{2}$, M. If there are five levels of any factor the coded doses would be of the form $-b$, $-a$, 0, a, b where $b > a$ and the corresponding actual doses would always be δ,

$$0,\ \left(1-\frac{a}{b}\right)\frac{M}{2},\ \frac{M}{2},\ \left(1+\frac{a}{2}\right)\frac{M}{2},\ M.$$

Here 0 against the actual doses stands for the control dose and M for the maximum dose of each factor to be tried in the experiment. The value of the scale parameters a and b can be obtained from relations involved in the design. The values of a and b have been worked out and are given along with the respective designs in the appendix.

4. Analysis of the Experiment

4.1. *Fitting of response surface*

The main purpose of analysis of such designs is to obtain a fit of the second degree response surface which can be written as

$$Y = b_0 + \Sigma b_i x_i + \Sigma b_{ii} x_i^2 + \sum_{j>i} b_{ij} x_i x_j$$

where x_i stands for the coded doses of the i-th factor present in the different design points. Assuming $\lambda_2 = 1$, the estimates of the constants be obtained from the following :

$$b_0 = D\{2\lambda_4^2(k+2)\Sigma Y - 2\lambda_4(\Sigma x_i^2 y)$$

$$b_i = \frac{\Sigma x_i y}{N}$$

$$b_{ij} = \frac{\Sigma x_i x_j y}{N\lambda_4}$$

$$b_{ii} = D[\{(k+1)\lambda_4 - (k-1)\}(\Sigma x_i^2 y - \Sigma y) - (\lambda_4 - 1)\sum_{j\neq i}(\Sigma x_i^2 y - \Sigma y)]$$

where $$D^{-1} = 2N\lambda_4\{(k+2)\lambda_4 - k\}$$

The variances and covariances of the above estimates of b's are

$$V(b_0)=2\lambda_4^2(k+2)D\sigma^2$$

$$V(b_1)=\frac{\sigma^2}{N}$$

$$V(b_{ij})=\frac{\sigma^2}{N\lambda_4}$$

$$V(b_{ii})=D\{(k+1)\lambda_4-(k-1)\}\sigma^2$$

$$CoV(b_{0i}b_{ii})=-2D\lambda_4\sigma^2$$

$$CoV(b_{ii}b_{ij})=-D(\lambda_4-1)\lambda^2$$

where σ^2 is the variance per plot.

Variance of the estimated response at points whose distance is 'ρ' from the origin is given by

$$V(\hat{y}_\rho)=V(b_0)+\rho^2\left\{1-\frac{2}{(k+2)\lambda_4-k}\right\}\sigma^2+\rho^4V(b_{ii})$$

4.2. *Estimation of* σ^2

After the response surface has been fitted, it is necessary to calculate the variance and co-variance of various constants fitted. All other quantities in the functions of variance and co-variance are known except σ^2. We have, therefore, to estimate this. Let there be 'r' replications of a design and m blocks per replication and each block with g plots. The analysis of variance table of the design would be as follows :

Analysis of Variance Table

Source of variation	*d.f.*		*M.S.*
Replications	$r-1$	$rm-1$	
Blocks within replications	$r(m-1)$		
Fitted constants	$\frac{k(k+3)}{2}$		
Deviation from fit	By subtraction		
Total:	$mrg-1$		

The M.S. for deviation from fit will provide an estimate of σ^2. In some of the designs, however, some treatment combinations are repeated within blocks. In those cases we may separate Pure Error from deviation of fit. The deviation from fit M.S., in these cases will be tested against Pure Error M.S. If the deviation from fit is not significant the pooled M.S. of Pure Error and deviation from fit will provide estimate of σ^2, otherwise the M.S. for Pure Error itself will provide the estimate of σ^2.

4.3. *Estimation of Optimum Combinations of Factors*

The main purpose of the response surface design is to obtain a set of combinations of the factors which provide the most profitable return. This can be worked out as given below :

Let p be the price per one unit of the yield of the crop on which the experiment has been conducted and q_i is the cost in rupees per one unit of input of the i-th factor in the conduct scale, the profit function obtained from the application x_i units of the i-th factor $(i=1, 2......k)$ assuming the second order response surface is

$$P=p\{b_0+\Sigma b_i x_i+\Sigma b_{ii} x_i^2+\Sigma_{j>i} b_{ij} x_i x_j\}-q_i x_i$$

The optimum combinations of the factor is the combination at which the profit is maximum. Thus, the solution of equations

$$\frac{dP}{dx_i}=0 \text{ for all } i=1, 2, 3, \ldots\ldots k$$

will provide the optimum combination.

Putting $\frac{q_i-pbi}{p}=li$, in these equations, they can be written as

$BX=L$,

$$\text{where } B=\begin{bmatrix} 2b_{11} b_{12} \cdots\cdots\cdots\cdots b_{1k} \\ b_{21} 2b_{22} \cdots\cdots\cdots\cdots b_{2k} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ b_{k1} b_{k2} \cdots\cdots\cdots\cdots 2b_{kk} \end{bmatrix}$$

$$X=(x_1 \ldots\ldots\ldots\ldots x_k)^T$$

and $$L=(l_1 \ldots\ldots\ldots\ldots l_k)^T$$

Thus the optimum combination vector X is, therefore, $X=CM$, where C is the matrix which is universe of B.

Summary

In the present paper several second order rotatable designs in 3 and 4 factors each at 3 or 5 levels split into blocks of equal size have been presented. The designs have been presented into coded doses which will be useful for fitting the response surface and also in actual doses which will be needed for the layout on the field. The scale of the design, i.e. the constants *a*, *b*, etc., are also given there. Analysis and working of optimum level combinations have also been given.

References

1. Box, G.E.P. and Hunter, J.S. : "Multifactor experimental designs for exploring response surface"—Ann. Math. Stat., 1957, 28.
2. Das, M.N. : "Construction of rotatable designs from factorial designs." Jour. Ind. Sec. Agri. Stat., Vol. 13, (1961)—169-194.
3. Das, M.N. and Gill, B.S. : "Block rotatable design for agricultural experimentation." Jour. Ind. Soc. Agri. Stat., under publication.

APPENDIX

Rotatable Designs in Blocks of Equal Sizes

Note : Each of the designs has been presented in two parts. In part (*a*) design with coded doses and in (*b*) the corresponding designs with actual doses are presented. In designs with actual doses the symbols *N, P, K* and *M* indicate the maximum levels of the factors that the experimenter may like to include in the experiment. The values of *a* and *b* have been worked. The minimum number of replications that may be needed are also given in the designs.

I. Designs with 3 levels for each of the factors

1.1 *Three Factors*

(*a*) *Coded Doses*

Block 1			*Block 2*			*Block 3*		
−a	a	0	−a	0	−a	0	−a	−a
−a	a	0	−a	0	a	0	−a	a
a	−a	0	a	0	−a	0	a	−a
a	a	0	a	0	a	0	a	a
0	0	a	0	a	0	a	0	0
0	0	−a	0	−a	0	−a	0	0
0	0	a	0	a	0	a	0	0
0	0	−a	0	a	0	a	0	0
0	0	0	0	0	0	0	0	0

a=1.5 λ_4=0.75, Minimum replications 2.

(*b*) *Actual Doses*

Block 1			*Block 2*			*Block 3*		
0	0	0.5K	0	0.5P	0	0.5N	0	0
0	P	0 5K	0	0.5P	K	0.5N	0	K
N	0	0.5K	N	0.5P	0	0.5N	P	0
N	0	0.5K	N	0.5P	K	0.5N	P	K
0.5N	0.5P	K	0.5N	P	0.5K	N	0.5P	0.5K
0.5N	0.5P	0	0.5N	0	0.5K	0	0.5P	0.5K
0.5N	0.5P	K	0 5N	P	0.5K	N	0.5P	0.5K
0.5N	0.5P	0	0 5N	0	0.5K	0	0.5P	0 5K
0.5N	0.5P	0.5K	0.5N	0.5P	0.5K	0.5N	0.5P	0 5K

1.2 *Four Factors*

(*a*) *Coded Doses*

Block 1				Block 2				Block 3			
a	a	0	0	a	0	a	0	a	0	0	a
−a	a	0	0	−a	0	a	0	−a	0	0	a
a	−a	0	0	a	0	−a	0	a	0	0	−a
−a	−a	0	0	−a	0	−a	0	−a	0	0	a
0	0	a	a	0	a	0	a	0	a	a	0
0	0	−a	a	0	−a	0	a	0	−a	a	0
0	0	a	−a	0	a	0	−a	0	a	−a	0
0	0	−a	−a	0	−a	0	−a	0	−a	−a	0
0	0	0	0	0	0	0	0	0	0	0	0

$a=1.5$ $\lambda_4=0.75$

(*b*) *Actual Doses*

Block 1				Block 2				Block 3			
N	P	0.5K	0.5M	N	0.5P	K	0.5M	N	0.5P	0.5K	M
0	P	0.5K	0.5M	0	0.5P	K	0.5M	0	0.5P	0.5K	M
N	0	0.5K	0.5M	N	0.5P	0	0.5M	N	0.5P	0.5K	0
0	0	0.5K	0.5M	0	0.5P	0	0.5M	0	0.5P	0.5K	0
0.5N	0.5P	K	M	0.5N	P	0.5K	M	0 5N	P	K	0.5M
0.5N	0.5P	0	M	0.5N	0	0.5K	M	0.5N	0	K	0.5M
0.5N	0.5P	K	0	0 5N	P	0.5K	0	0.5N	P	0	0 5M
0.5N	0.5P	0	0	0.3N	0	0.5K	0	0.5N	0	0	0 5M
0.5N	0.5P	0 5K	0 5M	0.5N	0 5P	0.5K	0.5M	0.5N	0.5P	0.5K	0.5M

1.3 *Four Factors*

(*a*) *Coded Doses*

Block 1				Block 2				Block 3				Block 4			
a	a	a	0	−a	−a	−a	0	a	a	0	a	−a	−a	0	−a
a	−a	−a	0	−a	a	a	0	a	−a	0	−a	−a	a	0	a
−a	a	−a	0	a	−a	a	0	−a	a	0	−a	a	−a	0	a
−a	−a	a	0	a	a	−a	0	−a	−a	0	a	a	a	0	−a
0	0	0	a	0	0	0	a	0	0	a	0	0	0	a	0
0	0	0	−a	0	0	0	−a	0	0	−a	0	0	0	−a	0
0	0	0	a	0	0	0	a	0	0	a	0	0	0	a	0
0	0	0	−a	0	0	0	−a	0	0	−a	0	0	0	−a	0
a	0	0	0	a	0	0	0	a	0	0	0	a	0	0	0
−a	0	0	0	−a	0	0	0	−a	0	0	0	−a	0	0	0
0	a	0	0	0	a	0	0	0	a	0	0	0	a	0	0
0	−a	0	0	0	−a	0	0	0	−a	0	0	0	−a	0	0
0	0	a	0	0	0	a	0	0	0	a	0	0	0	a	0
0	0	−a	0	0	0	−a	0	0	0	−a	0	0	0	−a	0
0	0	0	a	0	0	0	a	0	0	0	a	0	0	0	a
0	0	0	−a	0	0	0	−a	0	0	0	−a	0	0	0	−a

Block 5	*Block 6*	*Block 7*	*Block 8*
a 0 a a	−a 0 −a −a	0 a a a	0 −a −a −a
a 0 −a −a	−a 0 a a	0 −a −a a	0 a a −a
−a 0 −a a	a 0 a −a	0 −a a −a	0 a −a a
−a 0 a −a	a 0 −a a	0 a −a −a	0 −a a a
0 a 0 0	0 a 0 0	a 0 0 0	a 0 0 0
0 −a 0 0	0 −a 0 0	−a 0 0 0	−a 0 0 0
0 a 0 0	0 a 0 0	a 0 0 0	a 0 0 0
0 −a 0 0	0 −a 0 0	−a 0 0 0	−a 0 0 0
a 0 0 0	a 0 0 0	a 0 0 0	a 0 0 0
−a 0 0 0	−a 0 0 0	−a 0 0 0	−a 0 0 0
0 a 0 0	0 a 0 0	0 a 0 0	0 a 0 0
0 −a 0 0	0 −a 0 0	0 −a 0 0	0 −a 0 0
0 0 a 0	0 0 a 0	0 0 a 0	0 0 a 0
0 0 −a 0	0 0 −a 0	0 0 −a 0	0 0 −a 0
0 0 0 a	0 0 0 a	0 0 0 a	0 0 0 a
0 0 0 −a	0 0 0 −a	0 0 0 −a	0 0 0 −a

$a=\sqrt{\frac{8}{3}}$, $\lambda_4=\frac{8}{9}$

(*a*) *Actual Doses*

Block 1

N	P	K	0.5M
N	O	O	0.5M
O	P	O	0.5M
O	O	K	0 5M
0.5N	0.5P	0 5K	M
0.5N	0.5P	0.5K	O
0.5N	0.5P	0.5K	M
0.5N	0.5P	0.5K	O
N	0.5P	0.5K	0.5M
O	0. 5P	0.5K	0.5M
0.5N	P	0.5K	0.5M
0.5 N	O	0.5K	0.5M
0.5N	0.5P	K	0.5M
0.5N	0 5P	O	0.5M
0.5N	0.5P	0.5K	M
0.5N	0.5P	0 5K	O

Block 2

O	O	O	0.5M
O	P	K	0.5M
N	O	K	0.5M
N	P	O	0.5M
0.5N	0.5P	0.5K	M
0.5N	0.5P	0.5K	O
0.5N	0.5P	0.5K	M
0.5N	0.5P	0.5K	O
N	0.5P	0.5K	0.5M
O	0.5P	0.5K	0.5M
0.5N	P	0.5K	0.5M
0.5N	O	0.5K	0.5M
0.5N	0.5P	K	0.5M
0.5N	0.5P	O	0.5M
0.5N	0.5P	0.5K	M
0.5N	0.5P	0.5K	O

Block 3

N	P	0.5K	M
N	O	0.5K	O
O	P	0.5K	O
O	O	0.5K	M
0.5N	0.5P	K	0.5M
0.5N	0.5P	O	0.5M
0.5N	0.5P	K	0.5M
0.5N	0.5P	O	0.5M
N	0.5P	0.5K	0.5M
O	0.5P	0.5K	0.5M
0.5N	P	0.5K	0.5M
0.5N	O	0.5K	0.5M
0.5N	0.5P	K	0.5M
0.5N	0.5P	O	0.5W
0.5N	0.5P	0.5K	M
0.5N	0.5P	0.5K	O

Block 4

O	O	0.5K	O
O	P	0.5K	M
N	O	0.5K	M
N	P	0.5K	O
0.5N	0.5P	K	0.5M
0.5N	0.5P	O	0.5M
0.5N	0 5P	K	0.5M
0.5N	0.5P	O	0.5M
N	0.5P	0.5K	0.5M
O	0.5P	0.5K	0 5W
0.5N	P	0.5K	0.5M
0.5N	O	0.5K	0.5M
0.5N	0.5P	K	0.5M
0.5N	0.5P	O	0 5M
0.5N	0.5P	0.5K	M
0.5N	0.5P	0.5K	O

Block 5

N	0.5P	K	M
N	0.5P	O	O
O	0.5P	O	M
O	0.5P	K	O
0.5N	P	0.5K	0.5M
0.5N	O	0.5K	0.5M
0.5N	P	0.5K	0.5M
0.5N	O	0.5K	0.5M
N	0.5P	0.5K	0.5M
O	0.5P	0.5K	0.5M
0.5N	P	0.5K	0.5M
0.5N	O	0.5K	0.5M
0.5N	0.5P	K	0.5M
0.5N	0.5P	O	0.5M
0.5N	0.5P	0.5K	M
0.5N	0.5P	0.5K	O

Block 6

O	0.5P	O	O
O	0.5P	K	M
N	0.5P	K	O
N	0.5P	O	M
0.5N	P	0.5K	0.5M
0.5N	O	0.5K	0.5M
0.5N	P	0.5K	0.5M
0.5N	O	0.5K	0.5M
N	0.5P	0.5K	0.5M
O	0.5P	0.5K	0.5M
0.5N	P	0.5K	0.5M
0.5N	O	0.5K	0.5M
0.5N	0.5P	K	0.5M
0.5N	0.5P	O	0.5M
0.5N	0.5P	0.5K	M
0.5N	0.5P	0.5K	O

Block 7

0.5N	P	K	M
0.5N	O	O	M
0.5N	O	K	O
0.5N	P	O	O
N	0.5P	0.5K	0.5M
O	0.5P	0.5K	0.5M
N	0.5P	0.5K	0.5M
O	0.5P	0.5K	0.5M
N	0.5P	0.5K	0.5M
O	0.5P	0.5K	0.5M
0.5N	P	0.5K	0.5M
0.5N	O	0.5K	0.5M
0.5N	0.5P	K	0.5M
0.5N	0.5P	O	0.5M
0.5N	0.5P	0.5K	M
0.5N	0.5P	0.5K	O

Block 8

0.5N	O	O	O
0.5N	P	K	O
0.5N	P	O	M
0.5N	O	K	M
N	0.5P	0.5K	0.5M
O	0.5P	0.5K	0.5M
N	0.5P	0.5K	0.5M
O	0.5P	0.5K	0.5M
N	0.5P	0.5K	0.5M
O	0.5P	0.5K	0.5M
0.5N	P	0.5K	0.5M
0.5N	O	0.5K	0.5M
0.5N	0.5P	K	0.5M
0.5N	0.5P	O	0.5M
0.5N	0.5P	0.5K	M
0.5N	0.5P	0.5K	O

2. Designs with five levels of each of the factors.

2.1 *Two Factors*

(*a*) *Coded Doses*

Block 1		Block 2	
a	b	b	a
–a	–b	–b	–a
–b	a	a	–b
b	–a	–a	b
0	0	0	0

$a=0.601$, $b=1.451$, $a/b=0.414$, $\lambda_4=0.616$
Minimum replications 3.

(*b*) *Actual Doses*

Block 1		Block 2	
0.707N	P	N	0.707P
0.293N	O	O	0.293P
O	0.707P	0.707N	O
N	0.293P	0.293N	P
0.500N	0.500P	0.500N	0.500P

2.2 *Three Factors*

(*a*) *Coded Doses*

Block 1			Block 2		
a	a	a	–a	–a	–a
a	–a	–a	–a	a	a
–a	a	–a	a	–a	a
–a	–a	a	a	a	–a
b	0	0	b	0	0
–b	0	0	–b	0	0
0	b	0	0	b	0
0	–b	0	0	–b	0
0	0	b	0	0	b
0	0	–b	0	0	–b

$a=1{\cdot}118$, $b=1.581$, $a/b=0{\cdot}707$, $\lambda_4=1.25$. Minium replications 2.

(*b*) *Actual Doses*

Block 1			Block 2		
0.854N	0.854P	0.854K	0.146N	0.146P	0 146K
0.854N	0.146P	0.146K	0.146N	0.854P	0.854K
0.146N	0.854P	0.146K	0.854N	0,146P	0,854K
0.146N	0.146P	0.854K	0.854N	0.854P	0.146K
N	0.500P	0.500K	N	0.500P	0.500K
O	0.500P	0.500K	O	0.500P	0 500K
0.500N	P	0.500K	0.500N	P	0.500K
0.500N	O	0.500K	0.500N	O	0.500K
0.500N	0.500P	K	0.500N	0.500P	K
0 500N	0.500P	O	0.500N	0.500P	O

2.3 *Four Factors*

(*a*) *Coded Doses*

Block 1	Block 2	Block 3	Block 4	Block 5
a a a 0	−a −a −a 0	a a 0 a	−a −a 0 −a	a 0 a a
a −a −a 0	−a a a 0	−a −a 0 a	−a a 0 a	a 0 −a −a
−a a −a 0	a −a a 0	−a a 0 −a	a −a 0 a	−a 0 −a a
−a −a a 0	a a −a 0	a −a 0 −a	a a 0 −a	−a 0 a −a
0 0 0 a	0 0 0 a	0 0 a 0	0 0 a 0	0 a 0 0
0 0 0 −a	0 0 0 −a	0 0 −a 0	0 0 −a 0	0 −a 0 0
0 0 0 a	0 0 0 a	0 0 a 0	0 0 a 0	0 a 0 0
0 0 0 −a	0 0 0 −a	0 0 −a 0	0 0 −a 0	0 −a 0 0

Block 6	Block 7	Block 8	Block 9	Block 10
−a 0 −a −a	0 a a a	0 −a −a −a	b 0 0 0	b 0 0 0
−a 0 a a	0 −a −a a	0 a a a	−b 0 0 0	−b 0 0 0
a 0 a −a	0 −a a −a	0 a −a a	0 b 0 0	0 b 0 0
a 0 −a a	0 a −a −a	0 −a a a	0 −b 0 0	0 −b 0 0
0 a 0 0	a 0 0 0	a 0 0 0	0 0 b 0	0 0 b 0
0 −a 0 0	−a 0 0 0	−a 0 0 0	0 0 −b 0	0 0 −b 0
0 a 0 0	a 0 0 0	a 0 0 0	0 0 0 b	0 0 0 b
0 −a 0 0	−a 0 0 0	−a 0 0 0	0 0 0 −b	0 0 0 −b

$$\frac{a}{b}=\frac{1}{\sqrt{2}}=0.707$$

$$a=\sqrt{2} \qquad b=2 \qquad \lambda_4=\frac{4}{5}=0.80$$

(*b*) *Actual doses*

Block 1

0.854N	0.854P	0.854K	0.500M
0.854N	0.146P	0.146K	0.500M
0.146N	0.854P	0.146K	0.500M
0.146N	0.146P	0.854K	0.500M
0.500N	0.500P	0.500K	0.854M
0.500N	0.500P	0.500K	0.146M
0.500N	0.500P	0.500K	0.854M
0.500N	0.500P	0.500K	0.146M

Block 2

0.146N	0.146P	0.146K	0.500M
0.146N	0.854P	0.854K	0.500M
0.854N	0.146P	0.854K	0.500M
0.854N	0.854P	0.146K	0.500M
0.500N	0.500P	0.500K	0.854M
0.500N	0.500P	0.500K	0.146M
0.500N	0.500P	0.500K	0.854M
0.500N	0.500P	0.500K	0.146M

Block 3

0.854N	0.854P	0.500K	0.854M
0.146N	0.146P	0.500K	0.854M
0.146N	0.854P	0.500K	0.146M
0.854N	0.146P	0.854K	0.146M
0.500N	0.500P	0.146K	0.500M
0.500N	0.500P	0.146K	0.500M
0.500N	0.500P	0.854K	0.500M
0.500N	0.500P	0.146K	0.500M

Block 4

0.146N	0.146P	0.500K	0.146M
0.854N	0.854P	0.500K	0.146M
0.854N	0.146P	0.500K	0.854M
0.146N	0.854P	0.500K	0.854M
0.500N	0.500P	0.854K	0.500M
0.500N	0.500P	0.146K	0.500M
0.500N	0.500P	0.854K	0.500M
0.500N	0.500P	0.146K	0.500M

Block 5

0.854N	0.500P	0.854K	0.854M
0.854N	0.500P	0.146K	0.146M
0.146N	0.500P	0.146K	0.854M
0.146N	0.500P	0.854K	0.146M
0.500N	0.854P	0.500K	0.500M
0.500N	0.146P	0.500K	0.500M
0.500N	0.854P	0.500K	0.500M
0.500N	0.146P	0.500K	0.500M

Block 6

0.146N	0.500P	0.146K	0.146M
0.146N	0.500P	0.854K	0.854M
0.854N	0.500P	0.854K	0.146M
0.854N	0.500P	0.146K	0.854M
0.500N	0.854P	0.500K	0.500M
0.500N	0.146P	0.500K	0.500M
0.500N	0.854P	0.500K	0.500M
0.500N	0.146P	0.500K	0 500M

Block 7

0.500N	0.854P	0.854K	0.854M
0.500N	0.146P	0.146K	0.854M
0.500N	0.146P	0.854K	0.146M
0.500N	0.854P	0 146K	0.146M
0.854N	0.500P	0.500K	0.500M
0.146N	0.500P	0.500K	0.500M
0.854N	0.500P	0.500K	0.500M
0.146N	0.500P	0.500K	0.500M

Block 8

0.500N	0.146P	0.146K	0.146M
0.500N	0.854P	0.854K	0.146M
0.500N	0.854P	0.146K	0.854M
0.500N	0.146P	0.854K	0.854M
0.854N	0.500P	0.500K	0.500M
0.146N	0.500P	0.500K	0.500M
0.854N	0.500P	0.500K	0.500M
0.146N	0.500P	0.500K	0.500M

Block 9

N	0.500P	0 500K	0.500M
O	0.500P	0.500K	0.500M
0.500N	P	0.500K	0.500M
0.500N	O	0.500K	0.500M
0.500N	0.500P	K	0.500M
0.500N	0.500P	O	0.500M
0.500N	0 500P	0.500K	M
0.500N	0.500P	0.500K	O

Block 10

N	0.500P	0.500K	0.500M
O	0.500P	0.500K	0 500M
0.500N	P	0.500K	0.500M
0.500N	O	0.500K	0.500M
0.500N	0.500P	K	0.500M
0.500N	0.500P	O	0.500M
0.500N	0.500P	0.500K	M
0.500N	0.500P	0.500K	O